

## USING EMERGENT ORDER TO SHAPE A SPACE SOCIETY

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## INTRODUCTION

A fast-growing movement in the scientific community is reshaping the way that we view the world around us. The short-hand name for this movement is "chaos." Chaos is a science of the global, nonlinear nature of systems. The center of this set of ideas is that simple, deterministic systems can breed complexity. Systems as complex as the human body, ecology, the mind or a human society. While it is true that simple laws can breed complexity, the other side is that complex systems can breed order. It is the latter that I will focus on in this paper.

In the past, nonlinear was nearly synonymous with nonsolvable because no general analytic solutions exist. Mathematically, an essential difference exists between linear and nonlinear systems. For linear systems, you just break up the complicated system into many simple pieces and patch together the separated solutions for each piece to form a solution to the full problem. In contrast, solutions to a nonlinear system cannot be added to form a new solution. The system must be treated in its full complexity.

While it is true that no general analytical approach exists for reducing a complex system such as a society, it can be modeled. The technique involves a mathematical construct called phase space. In this space stable structures can appear which I use as analogies for the stable structures that appear in a complex system such as an ecology, the mind or a society. The common denominator in all of these systems is that they rely on a process called feedback loops. Feedback loops link the microscopic (individual) parts to the macroscopic (global) parts. The key, then, in shaping a space society, is in effectively using feedback loops. This paper will illustrate how one can model a space society by using methods that chaoticists have developed over the last hundred years. And I will show that common threads exist in the modeling of biological, economical, philosophical and sociological systems.

## PHASE SPACE VISUALIZATION OF DYNAMICAL SYSTEMS

In nonlinear science, modeling is performed on what are called "dynamical systems." This is a physical system that evolves in time according to well-defined rules. It is characterized by the rate of change of its variables as a function of the values of the variables at that time. Examples are Maxwell's equations, the Navier-Stokes equations, and Newton's equations of motion for a particle with suitably specified forces. Modeling of dynamical systems are done in "phase space." This mathematical space that has position ( $x$ ) as one axis and velocity ( $v$ ) as the other axis. However, sometimes scientists represent the phase space with just the spatial coordinates of the system to better visualize the dynamics. Each point in this space represents the complete behavior of the

system. For example, the position and velocity of a pendulum with one degree of freedom at any instant in time is in a two dimensional phase space. Notice that if we have a more complicated system the number of dimensions of the phase space is enormous. The reason being that the space is constructed by assigning coordinates to every independent variable (every degree of freedom requires 2 more dimensions in phase space). In fact, the dimension is taken to be infinite in the general hydrodynamic description.

Since all of the information about a system is stored in a point at one instant in time, the evolution of a system can be charted by the moving point tracing its path through phase space. The time evolution is often called a "trajectory" or an "orbit." The set of orbits originating from all possible initial conditions generates a "flow" in this space governed by a set of  $2n$  first-order coupled differential equations:

$$\frac{dx_i}{dt} = F(x_1, x_2, \dots, x_n; v_1, v_2, \dots, v_n), \quad i = 1, \dots, n$$

$$\frac{dv_i}{dt} = F(x_1, x_2, \dots, x_n; v_1, v_2, \dots, v_n), \quad i = 1, \dots, n$$

where  $n$  is the number of degrees of freedom and  $F$  is the rate of change in position and velocity. This change is added to the previous values of the position and velocity to get a new  $x, v$  point in phase space, i.e.:

$$x_{t+1} = x_t + F(x_1, x_2, \dots, x_n; v_1, v_2, \dots, v_n)$$

for position and

$$v_{t+1} = v_t + F(x_1, x_2, \dots, x_n; v_1, v_2, \dots, v_n)$$

for velocity, where  $t$  represents a particular time.

One advantage of thinking of states as points in space is that it makes *change* easier to watch. If some combination of variables never occur, then a scientist can simply imagine that that part of space is out of bounds. If a system behaves periodically, then the point will move around in a loop, passing through the same position in phase space again and again. The motion of a point in phase space must *always* be non-self-intersecting. This arises from the fact that a point in phase space representing the state of a system encodes all of the information about the system, including its future history, so that there cannot be two different pathways leading out of one and the same point.

Since scientists are usually interested in the long-term behavior of dynamical systems, what will be the nature of the motion after all of the short-lived motions have died out? For dynamical systems with friction or some other form of dissipation (i.e., a nonconservative system), the system will eventually approach a restricted region of the phase space called an attractor. This is the solution set of the dynamical system. If we know the structure of the attractor, then we can sensibly claim that we know all the important things about the solution of our differential equation.

## WHAT IS AN ATTRACTOR?

As the name implies, nearby initial conditions are “attracted;” the set of points that are attracted form the *basin of attraction*. A dynamical system can have more than one attractor, each with its own basin, in which case different initial conditions lead to different types of long-term behavior. The region in the  $2n$ -dimensional phase space occupied by the attractor, its “volume,” is, in general, very small relative to the amount of phase space.

The simplest attractor in phase space is a fixed point. Figure 1 shows the damped harmonic oscillator (Shaw, 1981):

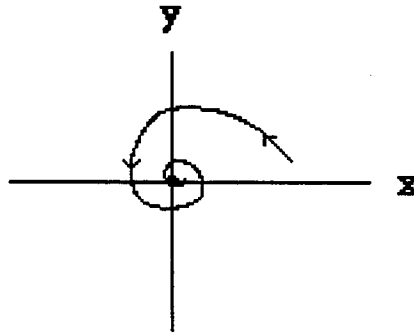


Figure 1. Damped harmonic oscillator. The equations describing motion are:

$$\frac{dx}{dt} = -y, \quad \frac{dy}{dt} = x - y$$

With fixed points, motion in phase space eventually stops; the system is attracted toward one point and stays there. Regardless of its initial position, the pendulum will eventually come to rest in a vertical position. Similarly, if a glass of water is shaken and then placed on a table, the water eventually approaches a state of uniform rest as its solution set. This is true despite the fact that the water’s phase space is initially virtually infinite in dimension.

Another example of an attractor is a periodic cycle called a limit cycle. Limit cycles represent a spontaneous sustained motion that is not often explicitly present in the equations describing the dynamical system. One example where this happens is the predator-prey system used in ecological models (Prigogine, 1980):

$$\frac{dX}{dt} = k_1AX - k_2XY$$

$$\frac{dY}{dt} = k_2XY - k_3Y$$

The variable  $X$  represents a prey population that uses product  $A$ ,  $Y$  represents a predator population that propagates at the expense of the prey, and  $k_i$  are constants. This relationship is usually referred

to the “Lotka-Volterra” equations. In phase space this system yields an infinite variety of close concentric orbits, as seen in figure 2. The different orbits, or periodic trajectories, correspond to different initial conditions and arise from the dynamics of the predator and prey reducing each other’s populations continuously.

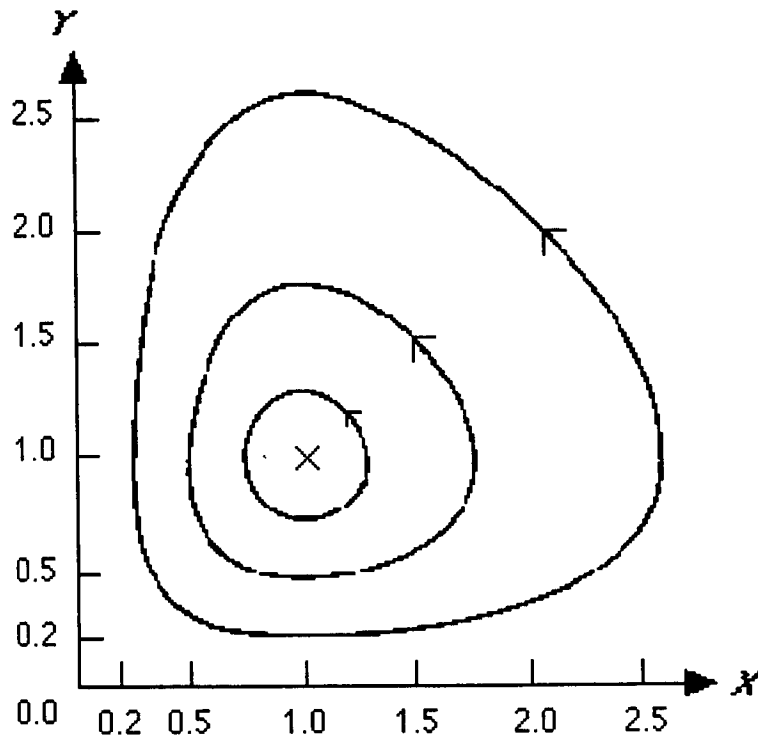


Figure 2. Periodic (Limit) cycle solutions of the Lotka-Volterra system for different initial conditions (Prigogine, 1980).

### STRANGE ATTRACTORS AND CHAOTIC BEHAVIOR

In dissipative systems, one can find attractors such as the two examples just mentioned (fixed and periodic). However, one can also find a *strange attractor*. The two examples just cited are oscillating systems without any forcing. When a forcing oscillator is added to the system, one has more dimensions in the phase space and the orbits converge to an object that is neither a fixed point nor a limit cycle. It is a strange attractor. Figure 3 is an example. This strange attractor depicts the chaotic behavior of a rotor, a pendulum swinging through a full circle, driven by an energetic kick at regular intervals.

A trajectory on a strange attractor exhibits most of the properties intuitively associated with random functions, although no randomness is ever explicitly added. The equations of motion are purely deterministic; the random behavior emerges spontaneously from the nonlinear system. This is often referred to as “deterministic chaos.” Over short times it is possible to follow the trajectory of

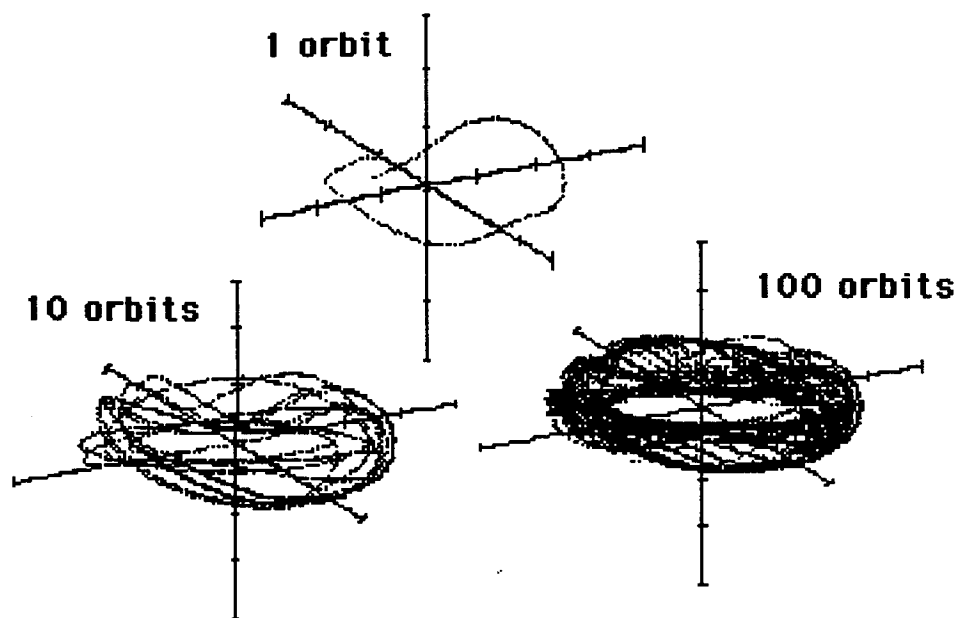


Figure 3. Snapshots in time of the Rotor strange attractor (Gleick, 1987).

each point, but over longer periods small differences in position are greatly amplified, so predictions of long-term behavior are impossible.

## BIFURCATIONS AND FEEDBACK LOOPS

I've now introduced phase space, attractors, strange attractors, and chaos. I will tie these ideas with what I call "emergent order" in this section.

In many dynamic systems there will be an instant in time when something as small as a slight fluctuation in density or a slight rise in temperature will be amplified by the iteration function  $F$  to a size so great that the system takes off in a new direction. This behavior is called a bifurcation, and it is the beginning of a qualitatively different solution of the underlying mathematical model. There are different types of bifurcations. In addition, bifurcations can combine with each other to produce a new state (steady, periodic, quasi-periodic, or chaotic) of the dynamic system. Figure 4 diagrams bifurcations. This figure was derived using a relationship for population growth called the "logistic map," which was first investigated by P. F. Verhulst in 1845. The Verhulst relationship is:

$$X_{n+1} = kX_n(1 - X_n)$$

where  $X$  represents the size of population, and  $k$  is a constant called the birthrate. As  $k$  increases, the system undergoes a bifurcation where there are two possible outcomes for population size and, therefore, the system periodically oscillates between them. When the birthrate increases again, we have four, eight, sixteen different population sizes. Finally, when  $k$  increases again, the total population size becomes chaotic. The chaotic zones are the dark regions filled with points. The bands of white are windows of stability where the system becomes stable and predictable again.

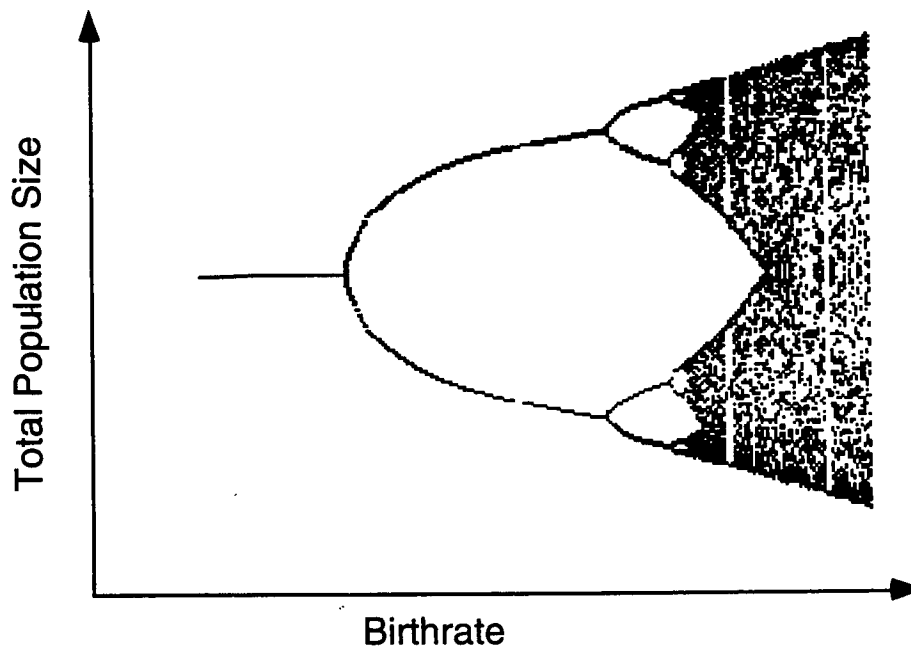


Figure 4. A model describing population growth called the Logistic Map (Stewart, 1989).

Over the course of time cascades of bifurcation points either cause the system to fragment itself, in what is called “period doubling toward chaos,” or it causes the system to stabilize a new behavior through a series of feedback loops. How are bifurcations and feedback loops related in a nonlinear model such as the one above?

To see how feedback loops are related, we can separate the Verhulst equation into two terms:  $kx_n$  and  $(1-X_n)$ . View the first as a “stretching” term and the other as a “folding” term. A property specific to all dissipative systems is that the volume of any set of initial conditions in phase space diminishes on the average in time. Chaoticists express this property by saying that “the flow contracts volumes in phase space.” The folding term is the reason for this property in the above nonlinear system. *The folding term is our feedback loop.*

The net result of bifurcations and feedback loops in a nonlinear and dissipative system is a process that links microscopic behavior with macroscopic behavior into an unpredictable, often symmetrical, beautiful form that can be characterized in phase space by a strange attractor. Strange attractors exhibit fractal microstructure. The most beautiful aspect of fractals is a quality of self-similarity which means that any section of the fractal, when blown up, reveals itself to be just as exquisitely detailed as was the larger picture from which it was taken.

Philosophically, this process means that we have a world that is interconnected, unpredictable, but with a subtle order present. Even what appears on the surface as disorder contains a high degree of implicit correlation. Sometimes this below-the-surface correlation can be triggered and it emerges to shape the system. I call this correlation *emergent order*. The following are some common ideas that I’ve noticed in systems exhibiting emergent order.

## COMMON THREADS IN SELF-ORGANIZED SYSTEMS

Self-organization operates on a wide range of levels. The three that I find most obvious are Minds, Ecologies/Evolution, and Free Markets. Common threads that run through all three are:

- 1) The order that emerges is unplanned and unpredictable.
- 2) Millions of things are operating under a simple set of rules while also operating in unique local conditions.
- 3) The rules evolve.
- 4) The individuals involved don't have to have knowledge or understanding of the whole system.
- 5) The systems regulate themselves by feedback loops.
- 6) The systems are as much *processes* as they are *systems* because they are continuously adapting to the fluctuations produced by the environment and as a result, matter/ life/ information is created and destroyed.
- 7) The systems are irreversible.
- 8) Intervention and attempts to control the systems will fail.

## SOCIAL AND ECONOMIC EVOLUTION

Since emergent order is unpredictable and unplanned, the best we can hope for in designing a society of productive people is to create relevant elements let the elements interact. If we try to control the detailed workings of these systems, we will interfere with its own logic and obstruct its self-ordering, rather than intelligently guiding it (Lavoie, 1989). What elements can we create?

The elements we can create are rules. These rules coordinate cooperation and agreement between unknown individuals (or companies) pursuing unknown purposes. Voluntary consent between the individuals is imperative (DiZerega, 1989). All participants within the system are equal in relationship to the rules. New rules may arise out of the self-organizing process in response to changes in the environment. The rules must be general and abstract and therefore, the particular details must be left to the individuals. (Hayek, 1975). Some rules could be the rules of property, the rules of contract, and the rules of law. In addition, the individuals could agree to certain rules of morals and rules of social convention. In a society based on division of labor and exchange, all individuals work to earn an income. So another rule could be that one's productivity (and hence their income) increases if one's effort increases.

The following are two social simulations with a resulting emergent order. Both simulations use very simple rules.

## EXAMPLE 1- EVOLUTION OF COOPERATION

In the early 1980's Robert Axelrod asked himself the question: Under what conditions will cooperation emerge in a world of egoists without central authority? In other words, in situations where each individual has an incentive to be selfish, how can cooperation ever develop (Barlow, 1991)? He framed this in a game that he called the "Prisoner's Dilemma." The game is that two accomplices to a crime are arrested and questioned separately. Either can renege against the other by confessing and hoping for a lighter sentence. However, if both confess, their confessions are not as valuable. If, on the other hand, both cooperate with each other by refusing to confess, then the district attorney can only convict them on a minor charge. In a collective sense it would be best for both of them to cooperate with each other. But if one has no conscience at all toward the other, and he confesses while the other one does not, then he still wins.

This game was quantified in a computer simulation and studied with methods of game theory. The problem was extended to apply to the same situation in sequential rounds so points were accumulated for each round. A computer tournament was staged in which participants sent in entries of their best strategies. The winning strategy was submitted by Professor Anatol Rapoport of the University of Toronto and was called: "Tit-for-Tat." The basic strategy was that each prisoner starts with one of the cooperative choices and then thereafter does what the other prisoner did on the previous round. The success of the strategy lies on its combination of being nice, retaliatory, forgiving and clear. These are good concepts to base any set of rules in setting up a space society. The feedback loop inherent in this strategy is a response to what the other prisoner did in the previous round.

## EXAMPLE 2- URBANIZATION

Another example of a simulation with very simple rules is that of urban evolution (Prigogine, 1980). One can use a variation of the logistic equation, mentioned previously, to characterize an urban region in terms of economic functions which are located at places called cities. The economic functions are dependent on populations. The efficiency of the feedback loop in which functions and populations relate depend on the increase of the population and the competition from other rival production units. When this model is run the appearance of an economic function works to destroy the initial uniformity of the population distribution by creating employment opportunities that concentrate the population at a point. These may be destroyed by the competition from similar but better developed economic functions. Some economic functions may develop in coexistence.

Figure 5 is a temporal sequence of an urbanization using the above model of an initial uniform region, in which four economic functions try to develop at each point in a network of fifty localities.

This type of model permits an estimation of the long-term consequences of decisions concerning elements such as transportation, investments, etc.

We see, that a model such as the above offers one an understanding of emergent order resulting from the choices of the many agents operating under similar constraints and pursuing their own goals. A complex structure such a space society can be shaped in this way.



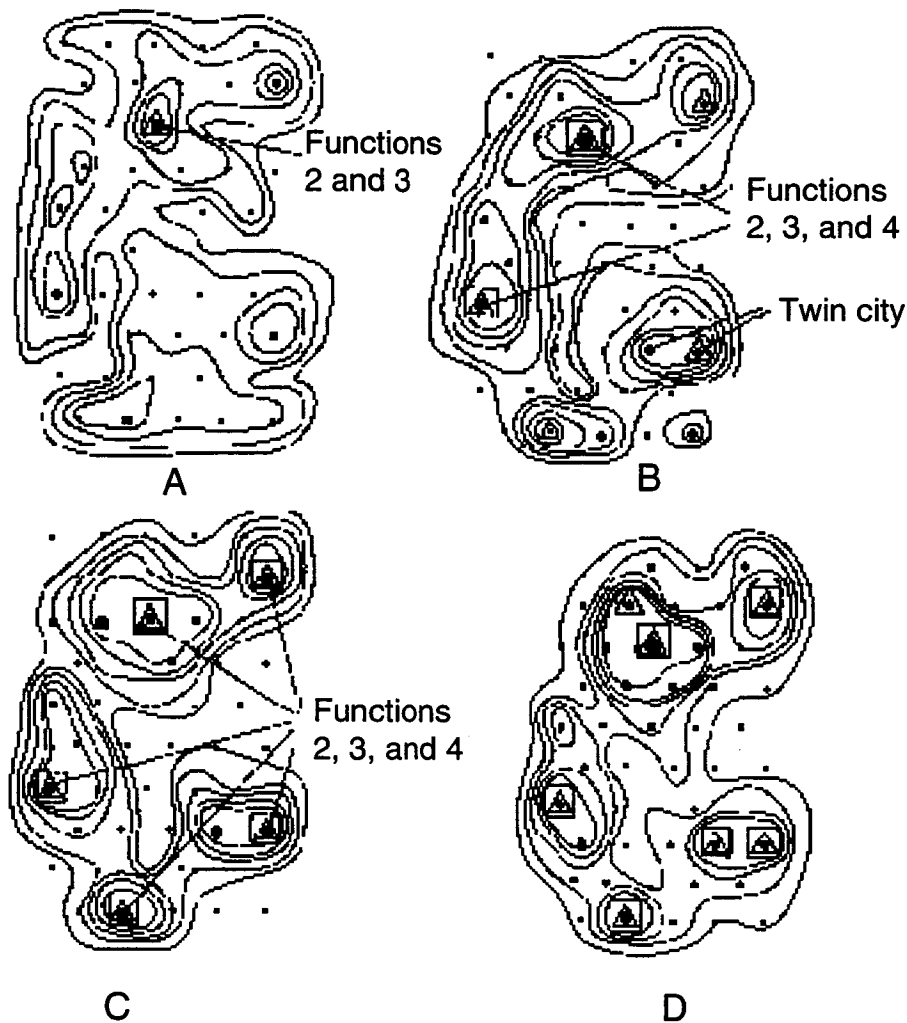


Figure 5. Urban evolution based on the logistic equation (Prigogine, 1980).

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## BIOGRAPHY

**Amara L. Graps** currently works at NASA Ames Research Center writing data reduction software for infrared astronomers and atmospheric scientists. Her specialties are in computational physics, computer graphics, and scientific data analysis. Her experience was gained from her current job at Ames, which she has held for six years, her previous job at the Laboratory for Atmospheric and Space Physics at the University of Colorado, and her job prior to that at the Jet Propulsion Lab.

At these jobs, she has analyzed data from NASA's Kuiper Airborne Observatory, the Voyager 2 spacecraft, the Pioneer Venus spacecraft, the Dutch's Infrared Astronomical Satellite (IRAS), and ground-based telescopes in Hawaii, California, and Arizona. The data are from Comet Halley, Supernova 1987a, Venus, Mars, Io, Saturn's and Uranus' rings, the Moon, Mercury, asteroids, Earth's atmosphere, protostars, galaxies and main-sequence stars. She got her first job (at JPL) in this field by volunteering her help to a woman who conducts a systematic search at Palomar Observatory to find asteroids. During her first trip there, she met several JPL scientists, who soon after offered her a job.

Ms. Graps earned her B.S. in Physics in 1984 from the University of California, Irvine while she was working at JPL, and her M.S. in Physics (with a Computational Physics option) in 1991 from San Jose State University while she has been at Ames.